

# Decoupling limit and throat geometry of non-susy D3 brane

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Recently it has been shown by us that, like BPS  $Dp$  branes, bulk gravity gets decoupled from the brane even for the non-susy  $Dp$  branes of type II string theories indicating a possible extension of AdS/CFT correspondence for the non-supersymmetric case. In that work, the decoupling of gravity on the non-susy  $Dp$  branes has been shown numerically for the general case as well as analytically for some special case. Here we discuss the decoupling limit and the throat geometry of the non-susy D3 brane when the charge associated with the brane is very large. We show that in the decoupling limit the throat geometry of the non-susy D3 brane, under appropriate coordinate change, reduces to the Constable-Myers solution and thus confirming that this solution is indeed the holographic dual of a (non-gravitational) gauge theory discussed there. We also show that when one of the parameters of the solution takes a specific value, it reduces, under another coordinate change, to the five-dimensional solution obtained by Csaki and Reece, again confirming its gauge theory interpretation.

AdS/CFT correspondence, in its original version [1] (see also [2, 3]), is an equivalence or a duality between two theories – one is a non-gravitational, conformally invariant and supersymmetric field theory in four dimensions (more precisely,  $D = 4$ ,  $\mathcal{N} = 4$  super Yang-Mills theory) and the other is a string theory (type IIB) or a gravitational theory in AdS space in five dimensions (times a five-dimensional sphere). It is holographic and is a strong-weak duality symmetry, in the sense, that when the field theory is strongly coupled the string theory is weakly coupled (given by supergravity) and vice-versa. This duality, is therefore, very useful to understand the strong coupling behavior of field theory by studying the weakly coupled string theory or supergravity. However, the theories on both sides of this duality are supersymmetric as well as conformally invariant and therefore not very realistic like QCD theory which is neither supersymmetric nor conformal. AdS/CFT correspondence has been extended for the less supersymmetric [4], non-conformal cases [5, 6] and even in other dimensions (other than three) [7] generally known as gauge/gravity duality (see [8] for a review). AdS/CFT type correspondence has also been studied for the non-supersymmetric (type 0) string theory solutions in [9].

There is no doubt that AdS/CFT type correspondence will be more useful if it can be understood for the non-supersymmetric, non-conformal case, where the associated field theory would be more like QCD and various strong coupling behavior of QCD can be understood by studying the dual gravity theory. However, the exact dual gravity theory which would correspond to QCD on the boundary is not known. But, it is clear that the relevant gravity solution must be non-supersymmetric. So, one could either start with a BPS brane-like solution and break the supersymmetry by compactification [10] or start directly with the non-supersymmetric brane-like solution of type II string theory [11]. Now,

for gauge/gravity duality to work in a brane-like gravitational background, there must exist a low energy or a decoupling limit for which bulk graviton must decouple from the brane. This can be shown either by calculating the graviton potential in the brane background which takes the form of an infinite barrier or by calculating the graviton absorption cross-section which vanishes in the decoupling limit. This is precisely what happens for the BPS  $Dp$  branes of type II string theory [12–15] and in [16], we have shown that exactly the same phenomenon occurs for the non-supersymmetric  $Dp$  brane solutions of the same theory as well. Usually it is assumed that gauge/gravity duality should work even for the non-supersymmetric case and the results in [16] clearly indicate that this is indeed true.

In this Letter we will consider the non-supersymmetric D3 brane solution of type IIB string theory and work out the decoupling limit more clearly. (An anisotropic non-susy D3 brane solution has been shown, by zooming into a particular space-time region, similar to the decoupling limit discussed in this Letter, to interpolate between  $AdS_5$  black hole,  $AdS_5$  soliton and a soft-wall gravity solution in [17].) In obtaining the decoupling limit for the non-susy case we will draw analogy from the BPS case and make sure that the decoupling limit goes over to the BPS D3 brane decoupling limit, when susy is restored. We will also show that the low energy excitations in the throat region and in the bulk get decoupled in the decoupling limit from the energy considerations. We then give the throat geometry which keeps the effective string action finite. Finally, by making an appropriate coordinate transformation, we show that the geometry is actually identical with the two parameter solution obtained previously by Constable and Myers [18]. We further show that when we fix one of the parameters and make another coordinate transformation the geometry reduces precisely to the one studied by Csaki and Reece [19]. Thus our

result justifies the gauge theory interpretation due to decoupling of gravity on the brane.

*Non-susy D3 brane* – The form of the non-supersymmetric D3 brane solution of type IIB string theory can be obtained by putting  $p = 3$  in eq.(2.11) of [16] which is

$$\begin{aligned} ds^2 &= F(\rho)^{-\frac{1}{2}} G(\rho)^{\frac{\delta}{4}} \left( -dt^2 + \sum_{i=1}^3 (dx^i)^2 \right) \\ &\quad + F(\rho)^{\frac{1}{2}} G(\rho)^{\frac{1+\delta}{4}} \left( \frac{d\rho^2}{G(\rho)} + \rho^2 d\Omega_5^2 \right), \\ e^{2\phi} &= G(\rho)^\delta, \quad F_{[5]} = \frac{1}{\sqrt{2}}(1 + *)Q \text{Vol}(\Omega_5) \end{aligned} \quad (1)$$

where, the functions  $F(\rho)$  and  $G(\rho)$  are given as,

$$\begin{aligned} F(\rho) &= G(\rho)^{\frac{\alpha}{2}} \cosh^2 \theta - G(\rho)^{-\frac{\beta}{2}} \sinh^2 \theta, \\ G(\rho) &= 1 + \frac{\rho_0^4}{\rho^4}, \end{aligned} \quad (2)$$

In the above the metric is given in the string frame and we have suppressed the string coupling constant  $g_s$  which is assumed to be small. The metric in (1) has  $\text{SO}(1,3) \times \text{SO}(6)$  symmetry and so, the solution is not of “black brane” type, rather, it is of BPS type. A “black brane” type solution should have  $\text{R} \times \text{SO}(3) \times \text{SO}(6)$  symmetry.  $F_{[5]}$  is the self-dual RR 5-form and  $Q$  is the charge of the non-susy D3 brane. Note from (2) that because of the form of  $G(\rho)$ , the solution has a naked singularity at  $\rho = 0$  and the physical region is given by  $\rho > 0$ . Further note that the solution is characterized by six parameters, namely,  $\alpha, \beta, \delta, \theta, Q$  and  $\rho_0$  of which  $\rho_0$  has the dimension of length,  $Q$  has the dimension of four-volume and others are dimensionless. Since  $e^\phi$  is the effective string coupling, the gravity solution (1) will remain valid only when the parameter  $\delta$  is less than or equal to zero and the radius of curvature (in string units) associated with the solution is very large. This latter restriction is satisfied in the decoupling limit when the charge of the brane is very large as discussed in the next section. The parameters of the solution mentioned above are not all independent as they must satisfy certain constraints for the consistency of the equations of motion. The constraints are,

$$\begin{aligned} \alpha &= \beta, \quad Q = 2\alpha\rho_0^4 \sinh 2\theta, \\ \alpha^2 + \delta^2 &= \frac{5}{2} \Rightarrow -\sqrt{\frac{5}{2}} \leq \delta \leq 0 \end{aligned} \quad (3)$$

Note that the above non-supersymmetric D3-brane is asymptotically flat. We can compare the non-susy D3 brane solution (1) with the BPS D3 brane solution. First of all, note that the non-susy D3 brane solution (1) contains three independent parameters ( $\rho_0, \theta, \delta$ ), whereas BPS D3 brane contains only one parameter (even the black D3 brane contains two parameters). Also BPS D3 brane is always charged under RR form-field, but the non-susy D3 brane can be chargeless by either putting

$\theta$  or  $\alpha$  (which is related to  $\delta$  by (3)) or both to zero (see (3)). Finally, we note that for non-susy D3 brane, the dilaton is in general not constant, however, it can be made constant by setting  $\delta$  to zero. But since  $\alpha$  and  $\delta$  are related by (3), they can not be simultaneously put to zero.

We can recover BPS D3 brane solution from the non-susy D3 brane solution given in (1) using a double scaling limit  $\rho_0 \rightarrow 0, \theta \rightarrow \infty$ , such that  $(\alpha/2)\rho_0^4(\cosh^2 \theta + \sinh^2 \theta) \rightarrow R^4 = \text{fixed}$ . Under this limit  $G(\rho) \rightarrow 1$ , and  $F(\rho) \rightarrow (1 + R^4/\rho^4)$  and  $Q \rightarrow 4R^4$  and then the solution (1) reduces to standard BPS D3 brane solution.

We would like to remark that as the solution given in (1) is not supersymmetric and has a naked singularity at  $\rho = 0$ , it is quite natural to ask whether the solution is stable under small classical perturbations. Unfortunately, the answer to this question with its full generality is not known. The study of stability under linear perturbations of non-supersymmetric space-time such as the Schwarzschild black holes both in four and higher dimensions has a long history and are given in [20]. These studies have been extended even for the globally naked singular solution in four and higher dimensions in [21, 22] and for the black  $p$ -brane solutions in higher dimensions in [23]. Keeping in mind the cosmic censorship hypothesis one might think that globally naked singular solution, such as the one discussed in this Letter, must be unstable under linear perturbations, but careful analysis given in [21], suggests that this apprehension is not always correct and there are stable nakedly singular solutions for certain physical boundary conditions. This has also been corroborated in the study of [24].

In [16], we have studied the dynamics of small classical graviton perturbations of scalar type (i.e., the perturbations are along the brane) and obtained a Schrödinger type equation satisfied by it. The analysis of the potential in this case suggests that at least for the scalar perturbations the background is stable. However, to claim that the space-time (1) is stable under linear perturbations we must also study the vector as well as the tensor perturbations with the proper boundary condition at the singularity [22]. This problem is currently under investigation.

*Decoupling limit* – As we know the decoupling limit is a low energy limit by which the fundamental string length  $\ell_s = \sqrt{\alpha'} \rightarrow 0$ . In this limit not only the interactions between the bulk theory and the theory living on the brane vanish, but also all the higher derivative terms in both the theories go to zero. Also as we have seen [16], in this limit, the classical scattering cross-section of a graviton moving in the brane background vanishes indicating that the bulk gravity possibly gets decoupled from the brane. This phenomenon is quite similar to the BPS case [15]. Now in order to find the decoupled geometry we make the following change of variables in analogy with BPS

D3 brane [1, 8],

$$\rho = \alpha' u, \quad \rho_0 = \alpha' u_0, \quad \alpha \cosh^2 \theta = \frac{L^4}{u_0^4 \alpha'^2} \quad (4)$$

along with  $\alpha' \rightarrow 0$ . Note that in the above  $u$  and  $u_0$  have the dimensions of energy and are kept fixed as we take  $\alpha' \rightarrow 0$ . Also  $L^4 = 2g_{YM}^2 N = \frac{R^4}{\alpha'^2}$  is a dimensionless parameter and remains fixed, where,  $g_{YM}^2 N$  is the 't Hooft coupling of the boundary theory. We would like to point out that in the limit, as  $\alpha' \rightarrow 0$ ,  $\rho_0 \rightarrow 0$  and  $\theta \rightarrow \infty$ , but, that does not imply that we have the BPS limit. This is because here  $\rho$  and  $\rho_0$  go to zero with the same scale and therefore,  $G(\rho)$  does not go to 1 as in the BPS limit. Furthermore, note from (4) and (3) that in the limit  $\alpha' \rightarrow 0$ , the charge has the value  $Q/\alpha'^2 \sim L^4 \gg 1$ . The last relation follows from the fact that the curvature of space-time in string units must be very very small for the supergravity description to remain valid. In [16] we found that the decoupling must occur also for small or even zero charge of the non-susy D3 brane, but we have not been able to find the explicit decoupling limit for these cases.

Now to justify the decoupling limit (4), we will see how in this limit we can keep the energy of a particle in the throat region in string units as well as that measured by an observer at infinity fixed [8]. Since  $g_{tt}$  as given in (1) is not constant these two energies will not be the same. So, if  $E_p$  denotes the energy of a particle as measured by an observer at a finite distance  $\rho$  from the brane and  $E$  denotes that of the same particle as measured by an observer at infinity, then they are related by a red-shift factor given by,

$$E = \sqrt{g_{tt}} E_p = F(\rho)^{-\frac{1}{4}} G(\rho)^{\frac{5}{8}} E_p \quad (5)$$

Under the decoupling limit (4) the functions  $G(\rho)$  and  $F(\rho)$  become

$$G(\rho) \rightarrow G(u) = 1 + \frac{u_0^4}{u^4} = \text{fixed}$$

$$F(\rho) \rightarrow F(u) = \tilde{F}(u) \frac{L^4}{\alpha u_0^4 \alpha'^2} \quad (6)$$

where,  $\tilde{F}(u) = G^{\frac{5}{2}}(u) - G^{-\frac{5}{2}}(u)$ . Then (5) takes the form

$$E = \tilde{F}(u)^{-\frac{1}{4}} \frac{\alpha^{\frac{1}{4}} u_0}{L} G(u)^{\frac{5}{8}} (\sqrt{\alpha'} E_p) \quad (7)$$

Therefore, if we keep  $\sqrt{\alpha'} E_p$  fixed, then  $E$  will remain fixed since the other quantities on the rhs of (7) are fixed in the decoupling limit. This gives a consistency check of the decoupling limit with the energy of an arbitrary excited string state and in turn implies (alongwith our observation in [16]) that the low energy excitations near  $\rho = 0$  and those in the bulk get decoupled in the decoupling limit. We can recover the results for the BPS D3

brane from here by putting  $u_0 \rightarrow 0$ . Now,  $G(u) \rightarrow 1$  and  $F(u) = \tilde{F}(u) \frac{L^4}{\alpha u_0^4 \alpha'^2} \rightarrow \frac{L^4}{\alpha'^2 u^4}$  and the energy relation (7) reduces to  $E = \frac{u}{L} (\sqrt{\alpha'} E_p)$ , precisely that of a BPS D3 brane [8].

*Throat geometry* – Here we will discuss the spacetime geometry for the non-susy D3 brane in the decoupling limit (4) we discussed in the previous section. In case of BPS D3 brane, the background becomes  $\text{AdS}_5 \times S^5$  in the corresponding decoupling limit. We have seen the form of the various functions in the decoupling limit in (6). The non-susy D3 brane solution (1) in the string frame becomes

$$ds^2 = \alpha' \frac{L^2}{u_0^2} \left[ \tilde{F}(u)^{-\frac{1}{2}} G(u)^{\frac{5}{4}} \left( -dt^2 + \sum_{i=1}^3 (dx^i)^2 \right) + \tilde{F}(u)^{\frac{1}{2}} G(u)^{\frac{1+\delta}{4}} \left( \frac{du^2}{G(u)} + u^2 d\Omega_5^2 \right) \right]$$

$$e^{2\phi} = g_s^2 G(u)^\delta, \quad F_{[5]} = \frac{2\sqrt{2}\alpha'^2}{\kappa} L^4 (1 + *) \text{Vol}(\Omega_5) \quad (8)$$

Here we have restored the string coupling constant  $g_s$ , and  $\kappa = \sqrt{8\pi G_{10}}$ , where  $G_{10}$  is the ten dimensional Newton's constant. The Yang-Mills coupling constant is related to  $g_s$  by  $g_{YM}^2 = 2\pi g_s$  and is independent of  $\alpha'$ . Also in the above we have redefined the coordinates  $(t, x^i) \rightarrow \frac{L^2}{\sqrt{\alpha} u_0} (t, x^i)$ , for  $i = 1, 2, 3$  and rescaled  $L^2 \rightarrow \sqrt{\alpha} L^2$ . The effective string coupling constant  $e^\phi = \frac{g_{\text{eff}}^2}{N} = g_s G(u)^{\frac{5}{2}} = \frac{g_{YM}^2}{2\pi} G(u)^{\frac{5}{2}}$  is also independent of  $\alpha'$ . We, therefore, claim (8) to be the throat geometry of non-susy D3 brane. It can be easily checked that in the BPS limit  $u_0 \rightarrow 0$  the above geometry reduces to  $\text{AdS}_5 \times S^5$ . The same geometry can also be obtained in the asymptotic limit, i.e., for  $u \rightarrow \infty$ . Now since there is decoupling of gravity on the non-susy D3 brane, this geometry must be dual to a QCD-like theory. To see that this is indeed true we will map the decoupled geometry (8) to the previously known geometry given by Constable and Myers [18] quite a while ago. In order to do that we redefine the function  $\tilde{F}(u)$  as  $\tilde{F}(u) = \hat{F}(u) G(u)^{-\frac{\alpha}{2}}$ , where,  $\hat{F}(u) = G(u)^\alpha - 1$ . The metric in the Einstein frame and the dilaton then take the forms,

$$ds^2 = \alpha' \frac{L^2}{u_0^2} \left[ \hat{F}(u)^{-\frac{1}{2}} G(u)^{\frac{\alpha}{4}} \left( -dt^2 + \sum_{i=1}^3 (dx^i)^2 \right) + \hat{F}(u)^{\frac{1}{2}} G(u)^{\frac{1-\alpha}{4}} \left( \frac{du^2}{G(u)} + u^2 d\Omega_5^2 \right) \right]$$

$$e^{2\phi} = g_s^2 G(u)^\delta \quad (9)$$

Then we make a coordinate transformation

$$u = \bar{r} \left( 1 + \frac{u_0^4}{4\bar{r}^4} \right)^{-\frac{1}{4}} \equiv \bar{r} \left( 1 + \frac{\omega^4}{\bar{r}^4} \right)^{-\frac{1}{4}}. \quad (10)$$

So the old harmonic function is modified to  $G(u) \rightarrow (1 + 2\omega^4/\bar{r}^4)^2$  and  $\hat{F}(u) \rightarrow (1 + 2\omega^4/\bar{r}^4)^{2\alpha} - 1 \equiv \bar{H}(\bar{r})$ . Therefore we find that, the metric and the dilaton (9) in this new coordinate, matches exactly with the Constable-Myers solution eqn.(2.1) of [18] if we identify the parameters as  $\alpha = \delta_{\text{CM}}/2$  and  $\delta = \Delta_{\text{CM}}/2$ , where we have denoted the Constable-Myers parameters with a subscript ‘CM’. The parameter relation  $\alpha^2 + \delta^2 = 5/2$  given in (3) then becomes  $\delta_{\text{CM}}^2 + \Delta_{\text{CM}}^2 = 10$  and is precisely the parameter relation given in Constable-Myers solution. We thus claim that the throat geometry in the decoupling limit of the non-susy D3 brane solution is nothing but the Constable-Myers two parameter solution. Since we already found that the bulk gravity gets decoupled for the non-susy D3-brane in the decoupling limit, so, the throat geometry must be the gravity dual of some QCD-like theory as discussed by Constable and Myers and our calculation justifies that.

We remark that the Constable-Myers two parameter solution have also been shown in [18] to arise from a suitable scaling limit of a non-susy D3 brane solution very similar to the decoupling limit we have discussed. But how that scaling ( $\beta$ ) is related to the physical low energy limit ( $\alpha' \rightarrow 0$ ) is not clear there. Identifying  $\beta = \cosh \theta$  in our solution we find from (4) that  $\beta = L^2/(\sqrt{\alpha}u_0^2\alpha')$  which indeed goes to infinity in the decoupling limit  $\alpha' \rightarrow 0$ . This clarifies why their scaled solution decouples gravity and represents a gravity dual of Yang-Mills type theory, actually a deformation of  $D = 4$ ,  $\mathcal{N} = 4$  super Yang-Mills theory, by breaking susy and conformal symmetry. This theory also contains massive fermions and scalars in the adjoint representation and has been shown to exhibit various QCD-like properties such as running coupling, confinement and mass gap in the glueball spectrum in certain range of parameters. Asymptotic behavior of the dilaton and the volume scalar determine the expectation values of the gauge invariant dimension four  $\text{Tr}(F^2)$  and dimension eight  $\text{Tr}(F^4 - (F^2)^2)$  operators in the gauge theory [18] in terms of the parameters of the gravity theory.

We will now show that the decoupled gravity background of non-susy D3 brane (9) describing QCD-like theory also studied by Constable and Myers can be reduced, in a special case, to another gravity background, supposed to describe infrared QCD-like theory which includes non-perturbative gluon condensate providing a natural IR cut-off for confinement, studied by Csaki and Reece [19]. For this we put  $\alpha = 1$ . So, by the constraint relation (3) we have  $\delta = \pm\sqrt{3/2}$ . We will take only the negative sign for  $\delta$  because as  $u \rightarrow 0$ , we want to keep the dilaton small. Also,  $\hat{F}(u)$  now takes the form  $\hat{F}(u) = G(u) - 1 = u_0^4/u^4$ . The metric and the dilaton

(9) then reduce in a new coordinate  $z = \frac{L^2}{u}$  to,

$$ds^2 = \alpha' \left[ \frac{L^2}{z^2} \left( G(z)^{\frac{1}{4}} (-dt^2 + \sum_{i=1}^3 (dx^i)^2) + \frac{dz^2}{G(z)} \right) + L^2 d\Omega_5^2 \right] \\ e^{2\phi} = g_s^2 G(z) - \sqrt{\frac{3}{2}} \quad (11)$$

where  $G(z) = 1 + \frac{z^4 u_0^4}{L^8} \equiv 1 + \frac{z^4}{z_0^4}$ . Note that the metric in (11) asymptotically ( $z \rightarrow 0$ ) has the form  $\text{AdS}_5 \times S^5$ . To cast the metric and the dilaton into the form of Csaki and Reece we need to go to another coordinate given by,

$$\hat{z} = z \left( \frac{1 + \sqrt{G(z)}}{2} \right)^{-\frac{1}{2}} \quad (12)$$

where  $G(z)$  is as given above. Now using (12) one can check the following identities,

$$\frac{2\sqrt{G(z)}}{1 + \sqrt{G(z)}} = 1 + \frac{\hat{z}^4}{\hat{z}_0^4} \equiv H(\hat{z}) \\ \frac{2}{1 + \sqrt{G(z)}} = 1 - \frac{\hat{z}^4}{\hat{z}_0^4} \equiv \tilde{H}(\hat{z}) \quad (13)$$

where in the above we have defined  $\hat{z}_0 = \sqrt{2}z_0$ . Now using (13), we can express (11) in terms of the harmonic functions  $H(\hat{z})$  and  $\tilde{H}(\hat{z})$  and find that they match exactly with the metric and the dilaton obtained by Csaki and Reece (eq.(3.10) and eq.(3.11) of [19]) as a gravity dual of a QCD-like theory. We have shown that this background is nothing but a special case of the throat limit of non-susy D3 brane. As we have seen in [16] that the bulk gravity indeed gets decoupled from the non-susy D3 brane in the decoupling limit, so our calculation justifies the gauge theory (or QCD-like theory) interpretation of this gravity background.

To conclude, in this Letter we have obtained the decoupling limit and the throat geometry of the well-known non-susy D3 brane solution of type IIB string theory. We would like to point out that the decoupling limit we have obtained in this Letter is only for the non-susy D3 brane with large RR charge. However, as we have seen in our previous work [16] that decoupling occurs also for the zero charge non-susy D3 brane. But we have not been able to obtain the decoupling limit for the zero charge case. Since BPS branes are always charged, we can take a guidance from it to obtain the decoupling limit for the charged non-susy D3 branes, as we have done in this paper, but there is no such analog for the chargeless case. However, it will certainly be interesting to understand the decoupling limit for the chargeless case and it remains an open problem. It would be interesting to use gauge/gravity duality to further explore the properties of the QCD-like theory that can be obtained from the throat geometry of the non-susy D3 brane background. For example, it may be possible to calculate the Wilson loop,



static as well as velocity-dependent quark-antiquark potential, screening length, monopole-antimonopole potential, jet quenching parameter and compare with the results already known for the susy  $\mathcal{N} = 4$  gauge theory.

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